

UNIVERSITY OF PENNSYLVANIA

THE "SPEEDOMAX" POWER-LEVEL
RECORDER

WILLIAM RUSSELL CLARK

A DISSERTATION

IN ELECTRICAL ENGINEERING

PRESENTED TO THE FACULTY OF THE MOORE SCHOOL OF THE
UNIVERSITY OF PENNSYLVANIA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR
OF SCIENCE IN ELECTRICAL ENGINEERING

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ASSOCIATE AIEE

POWER-LEVEL and attenuation measurements are increasing in importance and a larger number of these measurements are being made each day. They are of particular importance in the radio and telephone industries where it is essential to know the power-level variations in electrical circuits and acoustic devices, the gain of amplifiers, and the attenuation of filters or communication lines at various frequencies.

Since both power-level and attenuation are measured in decibels, methods for measuring them are similar. Manual methods of obtaining these measurements have been fairly well standardized but, while they have proved to be satisfactory for some laboratory experiments, they are not applicable for many production tests. Even in some laboratory cases the time allowable for making the measurements does not permit manual methods to be used and automatic recording is necessary.

An automatic recorder for recording power level or attenuation should faithfully record rapid or slow variations in power level with time. It should be independent of frequency variations within the range of interest and record only changes in power level. In general, the faster the recorder, the more faithful the recording, provided it does not overshoot. The recorder should not move in steps, but it should draw a smooth curve on rectangular co-ordinate paper in order to eliminate the need for interpolation at any point on the record.

As a result of the need for an automatic recording device of this type, the "Speedo-

max" power-level recorder was developed.

In the standard manual method of making precise power-level or attenuation measurements, the instrument under test is supplied with power of the desired frequency or frequencies and its output terminals are connected to the input terminals of an attenuator which is manually adjusted so that the deflection

Its requirements, therefore, are simple, and it is comparatively easy to design. Since precision attenuators are available, this method permits measurements to be made over wide decibel ranges with very accurate results. It eliminates two difficult problems, the designing of an a-c amplifier having reproducible characteristics over a large range of input voltage and the designing of a dynamic element with a logarithmic response.

The basic idea of the Speedomax power-level recorder was that an automatic method should be used to make power-level and attenuation measurements that would be similar to the standard manual method. The problem was to devise an electrical circuit that would operate a recording mechanism to do this. The electrical circuit must be insensitive to

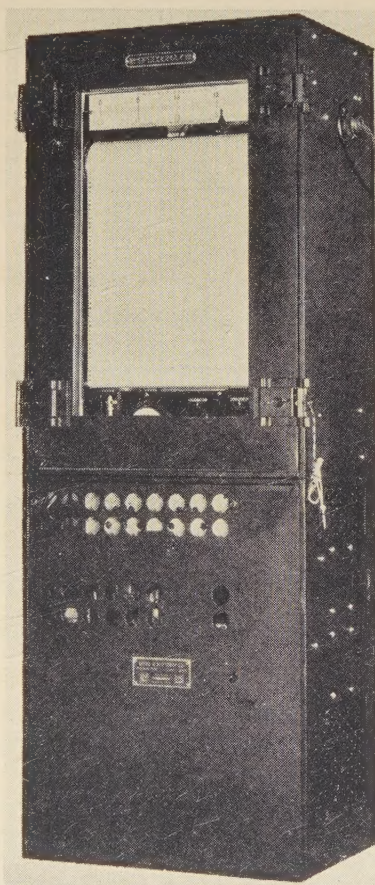


Figure 1 (above). The Speedomax power-level recorder (closed view)

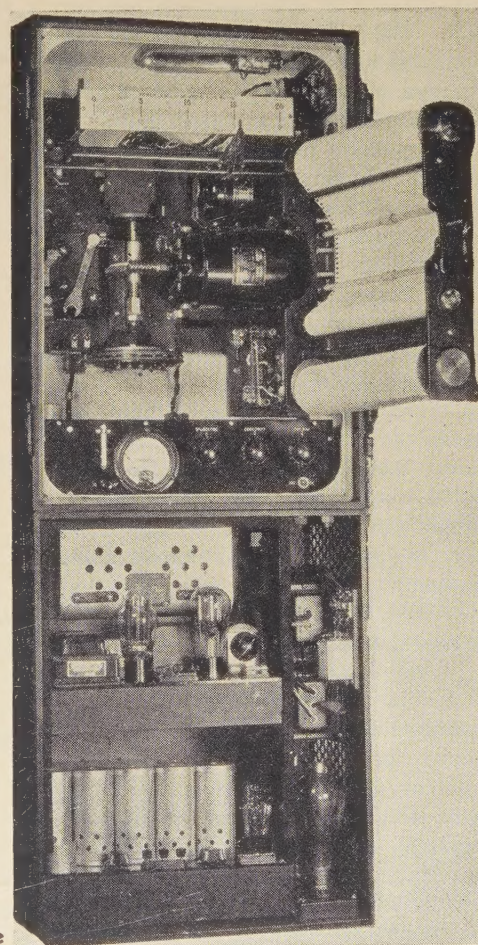


Figure 2 (right). The Speedomax power-level recorder (open view)

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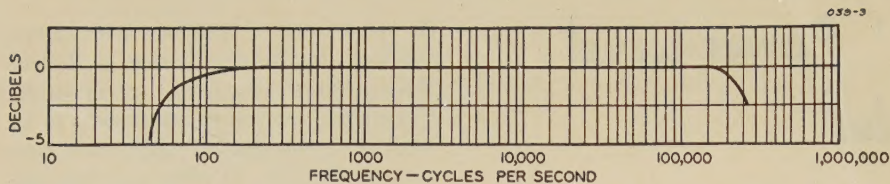
The author appreciates the suggestions of A. J. Williams, Jr., of the Leeds and Northrup Company in developing this instrument.

1. For all numbered references, see list at end of paper.

of an indicating device across the output terminals of the attenuator remains constant for variations in input power. This indicating device, which usually consists of an a-c amplifier, a rectifier, and a meter, operates at a constant power level.

frequency variations and only respond to changes in input power. The recording mechanism is similar to that of the high-speed recorder described in an article by A. J. Williams, Jr., in the 1938 AIEE TRANSACTIONS¹ and which is used to record small direct voltages.

The first instrument was completed in 1935, but the recorder described in this



paper is a later model which has an extended frequency range (figures 1 and 2). The scale is linear in decibels (logarithmic voltage scale). It has a range of 40 decibels above a zero level of 0.01 milliwatt with an input impedance of 135 ohms. It is designed to operate between 150 and 150,000 cycles (figure 3) and over this frequency range the recorder is accurate to one-half per cent of full scale (0.2 decibel). The speed of response of the recorder is such that the pen will traverse any part of the ten-inch chart and arrive at complete balance without overshooting in approximately one second. The paper is driven by a synchronous motor at speeds of either six inches a minute or twelve inches an hour. The recorder draws a smooth curve and operates direct from a 115-volt 60-cycle supply. No batteries are used.

A sample record made by the recorder is shown in figure 4. It was obtained by applying a 1,000-cycle voltage to the input terminals of a decade attenuator which was connected to the power-level recorder. By changing the settings of the attenuator the recorder pen was made to move across the scale. This record shows the accuracy of the recorder to be better than one-half per cent, the speed to be approximately one second for the largest unbalance, the dead range to be very small, and that the recorder draws a smooth curve.

General Description

In the operation of the recorder an unknown alternating voltage is impressed across the input terminals of a continuously adjustable ladder-type slide-wire attenuator (see the block diagram, figure 5). The output of the attenuator is amplified by an a-c amplifier and rectified by the detector tube. The d-c output voltage of the detector is amplified by the d-c amplifier. The output voltage of the d-c amplifier is introduced into the converter where it is opposed by a standard direct voltage. The resultant of the two voltages is impressed across the grids of the double-triode vacuum tube of the converter. If the two voltages do not balance each other, the resultant voltage will operate one of the triode units of the converter, and the direction of unbalance will determine

Figure 3. Frequency characteristic of the recorder

which unit operates. Each triode unit of the converter controls a thyatron, which in turn controls the direction of rotation of the slide-wire motor. Consequently, when one-half of the double triode operates one thyatron is turned on, the motor revolves and moves the

slide-wire attenuator and the pen which are mechanically coupled to it. When the alternating voltage applied to the input terminals of the recorder changes, the motor revolves and moves the attenuator to a position that will restore the original voltage level of the a-c amplifier and balance the recorder without overshooting. Since it is desirable to avoid the use of batteries in this instrument, the constant voltages necessary for operation of the amplifiers and detectors are obtained from a regulated power supply.

In this recorder the balance point is not approached at constant speed, but the speed of rebalancing increases in proportion to the amount of unbalance. Therefore, large changes in power levels

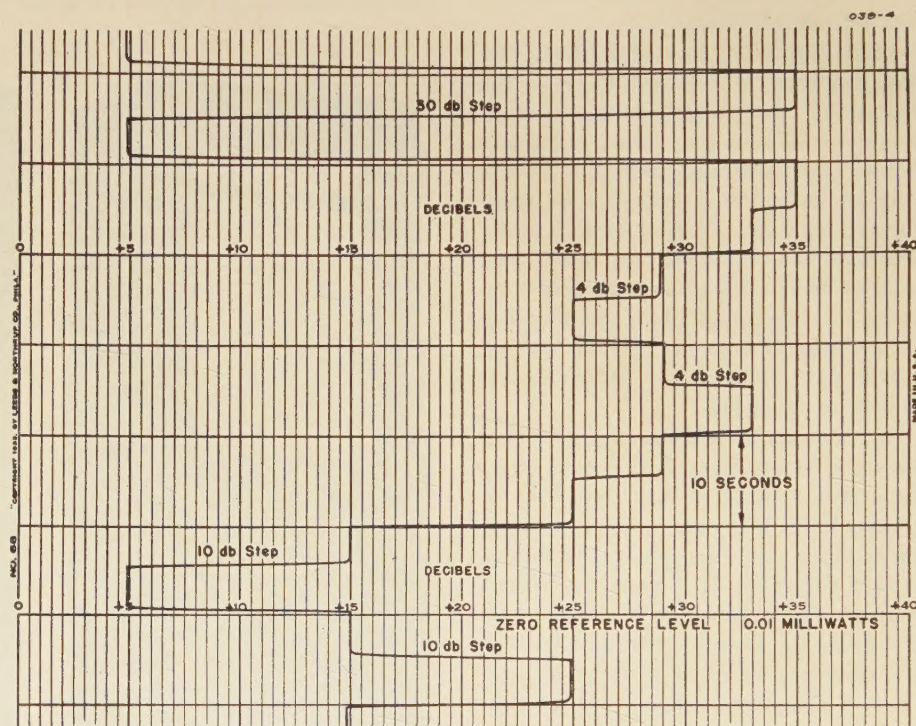


Figure 4. Chart made by recorder

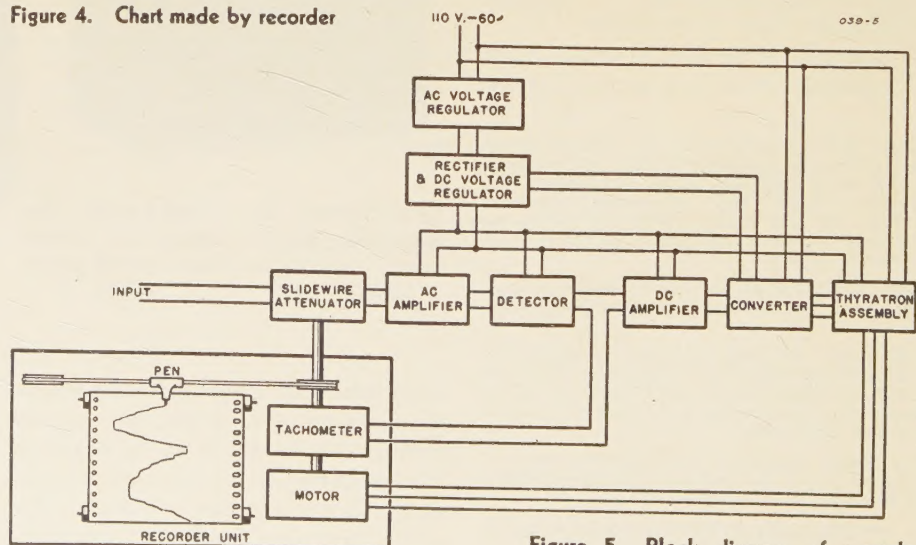
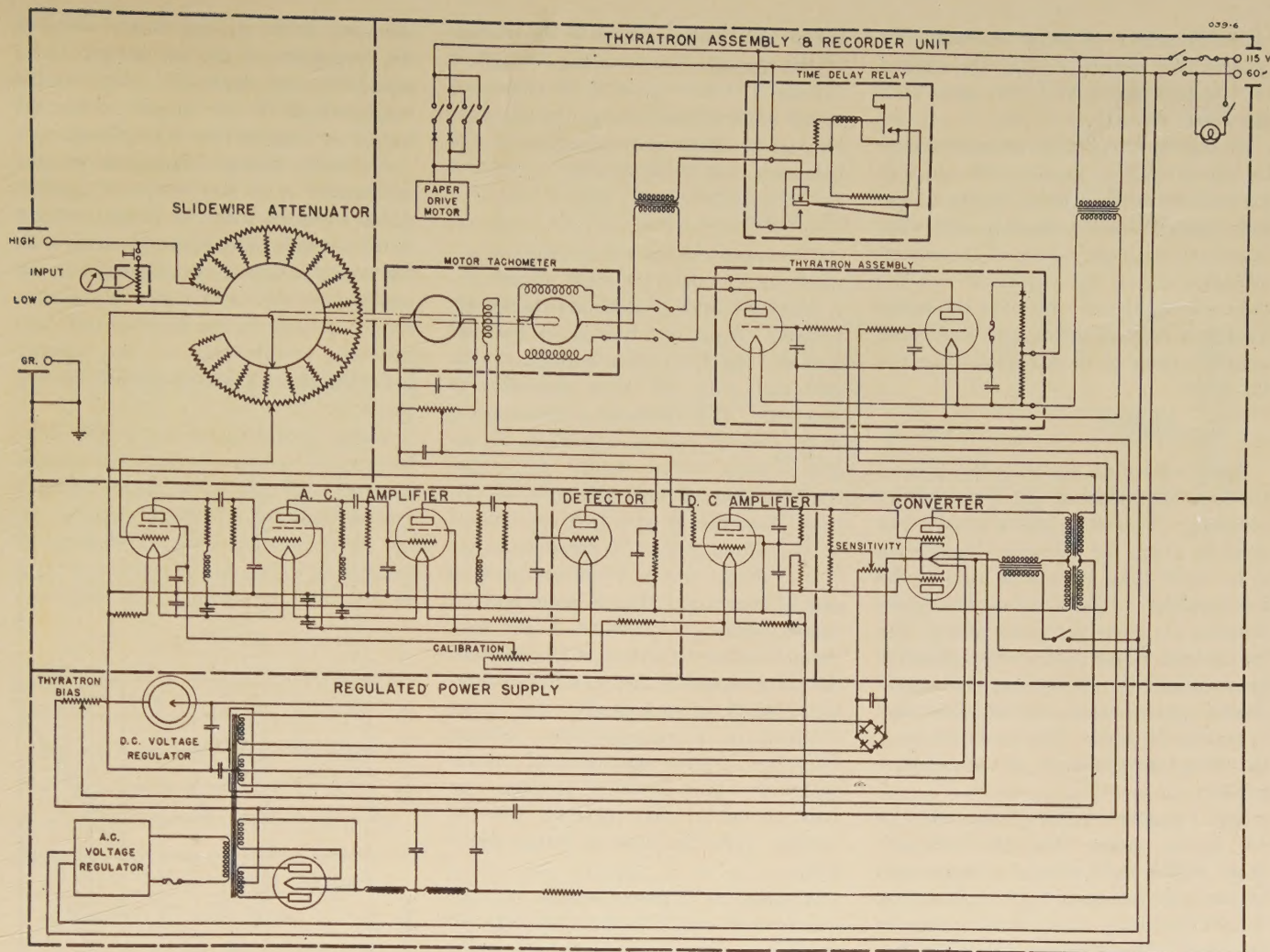


Figure 5. Block diagram of recorder



will be recorded approximately as fast as small changes.

Detailed Description of Component Parts

SLIDE-WIRE ATTENUATOR

The decibel range and the characteristic impedance of the recorder are determined by the slide-wire attenuator. This type of attenuator is preferable to a step-by-step attenuator because its attenuation is continuously adjustable and, therefore, enables more accurate measurements to be obtained. The attenuator in this recorder has a range of 40 decibels with a characteristic impedance of 135 ohms. Slide-wire attenuators have been constructed with other decibel ranges, and with various other characteristic impedances.

The accuracy of the recorder depends on the linearity and stability of this attenuator. The linear decibel scale is obtained by using a ladder-type attenuator which is a continuously variable unbalanced π -section slide-wire attenuator (figures 6 and 7). The output of the

Figure 6. Schematic wiring diagram of the recorder

attenuator operates into the grid circuit of a vacuum tube, which, because of its high impedance, introduces no attenuation error. Since the moving contacts of the attenuator are in this high-impedance circuit, they cause no trouble in the operation of the recorder.

The series coils are in the form of a uniform tapped slide-wire which is divided into equally spaced sections having the same resistance per section. A shunt coil is connected to each tap on the slide-wire.

In designing a slide-wire attenuator, which has a specified attenuation, the number of sections is determined by the allowable attenuation error and is independent of the characteristic impedance (see the development in the appendix). The values of the attenuation in decibels at successive shunt coils lie on a curve which is linear, but the variation in the attenuation between shunt coils is not linear (figure 8). This error, which can be

made negligible by using a sufficient number of sections, can be readily calculated as is shown in the appendix. In the graphs of figure 9, which were obtained from equations 9 and 10 of the appendix, the location of the point in each attenuator section at which the attenuation error is greatest ($n_{\max}$) and the value of this error ($D_{\max}$) are plotted against sections per decibel (S/A). These graphs show that for values of S/A greater than $1/4$ the maximum error is small and occurs very close to the mid-point of each section. In this particular recorder, the range was 40 decibels and by using a 15-section slide-wire attenuator, it was made linear in decibels to one-eighth per cent (see figure 8).

These graphs apply to all unbalanced slide-wire attenuators and if the allowable error and the total attenuation is known, then the number of sections can readily be obtained. The slide-wire resistance and the shunt coils can then be calculated, to obtain any characteristic impedance.

The slide-wire attenuation must be constant over the entire frequency range

of the recorder. In order to obtain this feature, the slide-wire is reverse wound on a hard-rubber rod and the shunt coils are woven resistors.

Frequency errors of the attenuator may be calculated from equations 3 and 6 of the appendix if the phase angles of the slide-wire and shunt coils are known. The calculated frequency error of the experimental model of the attenuator was less than one-eighth per cent over the entire frequency range of the recorder and measurements made on the attenuator checked this figure.

A-C AMPLIFIER

As stated before, the recorder operates to keep the input of the a-c amplifier constant. Therefore, the lowest power level at which the recorder will operate is no lower than the power level of the a-c amplifier. The greater the amplification, the lower the power level. The lowest level of the experimental model is approximately ten microwatts and a gain control is provided so that this level may be readily adjusted. The maximum overall voltage amplification of the amplifier is 300.

The frequency range of the recorder can be no greater than the frequency range of the a-c amplifier. In general, by carefully designing this amplifier, the desired frequency characteristics may be obtained. The experimental model has a three-stage resistance-capacitance coupled amplifier (figure 6) which uses screen-grid tubes. By reducing unwanted

feedback, as they cause an error in the reading of the recorder. In order not to affect the speed of the recorder, the time constants of certain circuits in the amplifier had to be small in comparison to the balancing time of the recorder.

DETECTOR

The output of the a-c amplifier is applied to the detector tube (figure 6). A triode detector is used and its accompanying circuit is adjusted so that increasing the alternating voltage on the grid will cause the tube gradually to overload. This produces a symmetrical plate current versus logarithm of the grid voltage curve (figure 10). The characteristic of the slide-wire, as already described, is such that any change in its position gives a logarithmic change in its voltage output, therefore figure 10 can be considered also as plate current versus slide-wire position. For the normal balanced position of the recorder the plate current is at the center of symmetry which is one-half the value of the overload or maximum plate current. Therefore, a given unbalance of the recorder in either direction produces the same change of plate current, and the recorder runs the same in either direction.

A capacitor is placed across the load resistance in the plate circuit in order to obtain a steady direct voltage by smoothing out the pulsating direct plate current flowing in the resistor. The capacity of this capacitor is limited to a value which

the rest of the recorder independent of the frequency of the a-c supply being measured, and dependent only on the magnitude of the a-c supply (within the frequency range of the a-c amplifier).

A direct voltage generated by the tachometer of the recording unit is introduced in series with the output voltage of the detector tube (figure 6). This voltage which is directly proportional to the speed of the slide-wire motor opposes the output voltage of the detector tube and prevents overshooting of the balance point due to the inertia of the moving parts.

When an unbalance occurs the slide-wire motor speeds up until the voltage generated by the tachometer becomes greater than the unbalance output voltage of the detector tube. This shuts off the driving torque on the motor and turns on the braking torque. The slide-wire

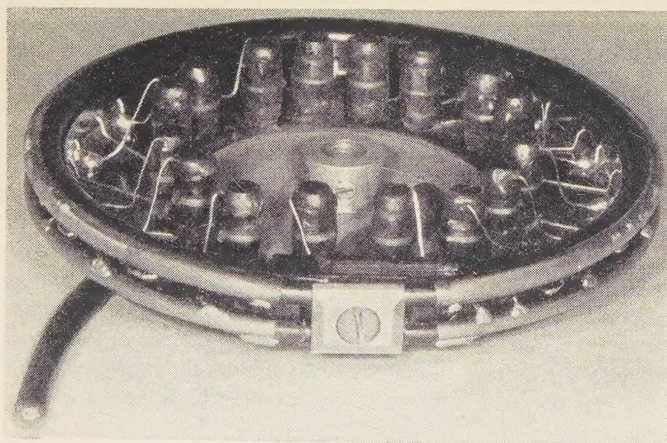


Figure 7. The slide-wire attenuator

feedback to a negligible amount and using broadly tuned circuits in addition to the resistance-capacitance coupling, the frequency characteristic of the a-c amplifier is made flat to within 0.1 decibel from 150 cycles to 150 kilocycles (figure 3).

Precautions are taken to keep any stray alternating voltages out of the a-c ampli-

fier, as they cause an error in the reading of the recorder. In order not to affect the speed of the recorder, the time constants of certain circuits in the amplifier had to be small in comparison to the balancing time of the recorder.

Converting the alternating voltage of the a-c amplifier to a direct voltage makes

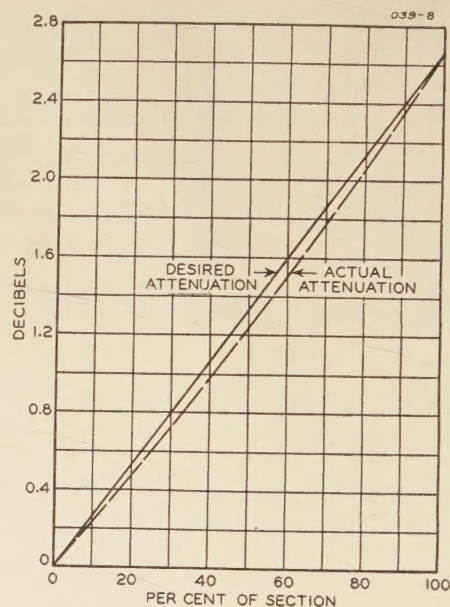


Figure 8. Accuracy graph of an attenuator section

motor slows down until the tachometer voltage becomes less than the unbalance output voltage of the detector tube, then the driving torque is again applied. This process is repeated until the recorder is brought to the balanced position which it attains without overshooting. During the balancing period the torque on the motor reverses, but the direction of rotation does not change.¹

The unbalanced voltage applied to the d-c amplifier is, therefore, alternately positive and negative during the balancing cycle, but is zero when the recorder is balanced.

For a given per cent unbalance of the

recorder, the magnitude of the d-c output voltage of the detector is greater for a large decibel range than for a small one. The magnitude of the braking voltage generated by the tachometer should approach the magnitude of the d-c output voltage of the detector. Too much tachometer voltage will make the recorder slow, while not enough voltage will allow the recorder to overshoot. If the speed of the recorder is to be independent

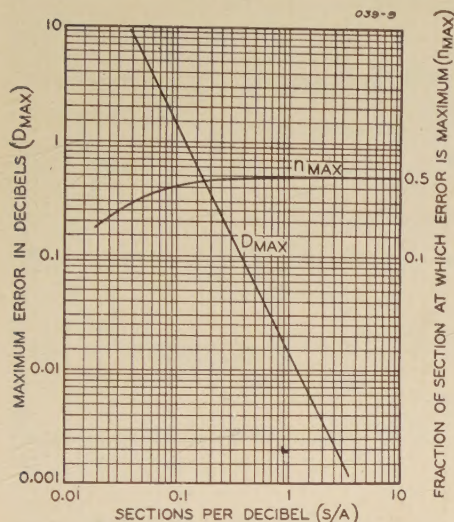


Figure 9. Maximum attenuation error and its location versus sections per decibel

of the decibel range, then the amount of tachometer voltage necessary to prevent overshooting will depend on the decibel range of the recorder; the larger the range, the larger the voltage required.

The kinetic energy of the moving system is proportional to the square of the

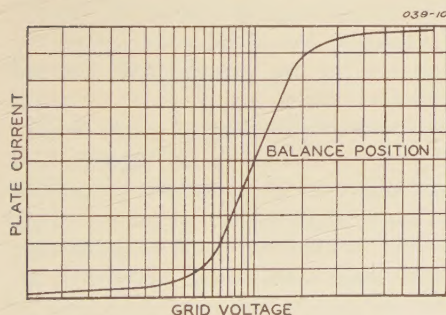


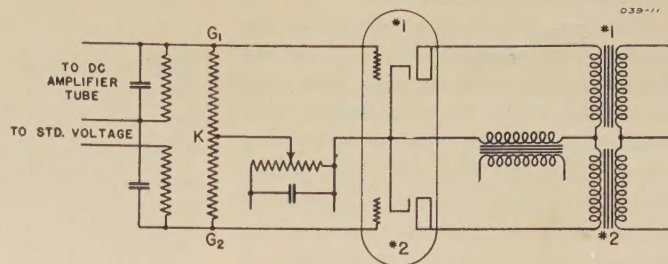
Figure 10. Plate current versus grid voltage curve of the detector

speed, but the energy which can be absorbed by the braking of the motor is proportional directly to the remaining distance to the balance point. In order to obtain maximum recorder speed without overshooting, these two energies should be equal throughout the entire decelerating period. If this is to be the

case, then either the damping voltage of the tachometer should be proportional to the square of the speed or else the output voltage of the detector tube should be proportional to the square root of the unbalance.¹

In this recorder the tachometer voltage is a linear function of the speed, but each

Figure 11. The converter



half of the detector plate current versus logarithm of the grid voltage curve (figure 10) besides being symmetrical is approximately a square-root relationship. Thus the above requirement is satisfactorily fulfilled and the greatest speed is obtained.

D-C AMPLIFIER

In order to obtain the desired sensitivity for the recorder, it is necessary to have a d-c amplifier since, for a small unbalance of the recorder, the voltage swing across the load resistance of the detector tube is not of sufficient magnitude to operate the converter. A single-stage screen-grid amplifier (figure 6) which has a voltage amplification of five is sufficient for recorder ranges as low as ten decibels. Lower ranges require additional amplification. A capacitor is placed across the load resistance in the plate circuit in order to smooth out any pulses which may appear in the voltage across this resistance. The time constant of this resistance-capacitor combination must also be kept low.

THE CONVERTER

The next problem is to utilize this d-c output voltage to operate the thyratrons and drive the slide-wire motor. It is not a simple one, because the d-c output of the d-c amplifier is 90 volts above ground, while the thyratrons are at ground potential. Insulating transformers must be used, but in order to use insulating transformers, the output voltage of the d-c amplifier must be transformed into an alternating voltage or used to control an alternating voltage. The alternating voltage must operate one thyatron when the output of the d-c amplifier increases, and must operate the other thyatron

when the output of the d-c amplifier decreases. The converter does all of this.

The output voltage of the d-c amplifier goes to the converter (figures 6 and 11) where it is opposed by a standard direct voltage obtained from the regulated power supply. The difference between these two voltages is applied across the

two resistances in the grid circuits of a double triode.

The grids of the double triode are biased so that when the two voltages are balanced neither section of the double triode will pass plate current.

Each plate of the double triode is connected through its own plate transformer to the same 60-cycle voltage supply. Therefore, the alternating voltage between the plate and cathode of each triode section is of the same magnitude and phase.

Referring to figure 11, if the output voltage of the d-c amplifier becomes less than the standard voltage, then G_1 will become positive with respect to K and triode number 1 will pass plate current, but since G_2 becomes negative with respect to K , triode number 2 will not pass plate current. If the output voltage of the d-c amplifier becomes greater than the standard voltage, then G_2 will become positive with respect to K , and triode number 2 will pass plate current, but since G_1 becomes negative with respect to K , triode number 1 will not pass plate current. When either triode passes plate current, a half-wave 60-cycle current flows through the primary of its plate transformer and produces a peak voltage wave across the transformer secondary. This voltage wave is, of course, the derivative of the current wave (figure 12) and is positive only one-fourth of a cycle. This is a desirable feature of the circuit, since the phasing between this voltage, which is applied to the grid of the thyatron, and the 60-cycle sine-wave plate voltage does not have to be adjusted as accurately as is generally necessary when sine-wave voltages are used on the grid as well as on the plate of the thyatron. It also insures that the

thyatron will be turned on at the beginning of the positive half of its plate voltage cycle, and thus obtains maximum power output for each cycle during operation.

Since only one section of the double triode can operate at one time, only one thyatron can be turned on at one time. In this way the power, which drives the slide-wire motor, is controlled.

The filament and bias voltages are obtained from separate windings on the power-pack transformer, since they must be insulated from the main filtered regulated supply. The impedance between these windings and the high-voltage windings of this transformer should be considerable, in order to minimize the leakage currents which flow through the grid resistances of the converter tube. In order further to reduce the effect of those currents which get into the grid resistances of the tube, the impedance from G_2 to ground was made the same as the impedance from G_1 to ground (figure 11). Now the leakage currents divide equally and their resulting effect has been reduced to zero.

Up to this point in the recorder the power supply has been obtained from regulated sources. The plate voltage for the converter tube is, however, not regulated, but it is obtained from the same source as that from which the plate voltage for the thyratrons is obtained. This insures correct phasing between the voltage ap-

plied to the thyatron grids and that used on their plates. Normal line-voltage variations at this point in the recorder will cause no error in its operation.

THYRATRON ASSEMBLY AND RECORDING UNIT

The output of the converter, as stated before, is applied to the grids of the

the meantime the cathodes are held more or less up from their normal zero value as is desired.

The record is made by a pen with ink on a chart ten inches wide and as long as desirable (up to 40 yards). A 12-inch length of chart is exposed. There are two paper speeds in the experimental model, 6 inches per minute or 12 inches per hour.

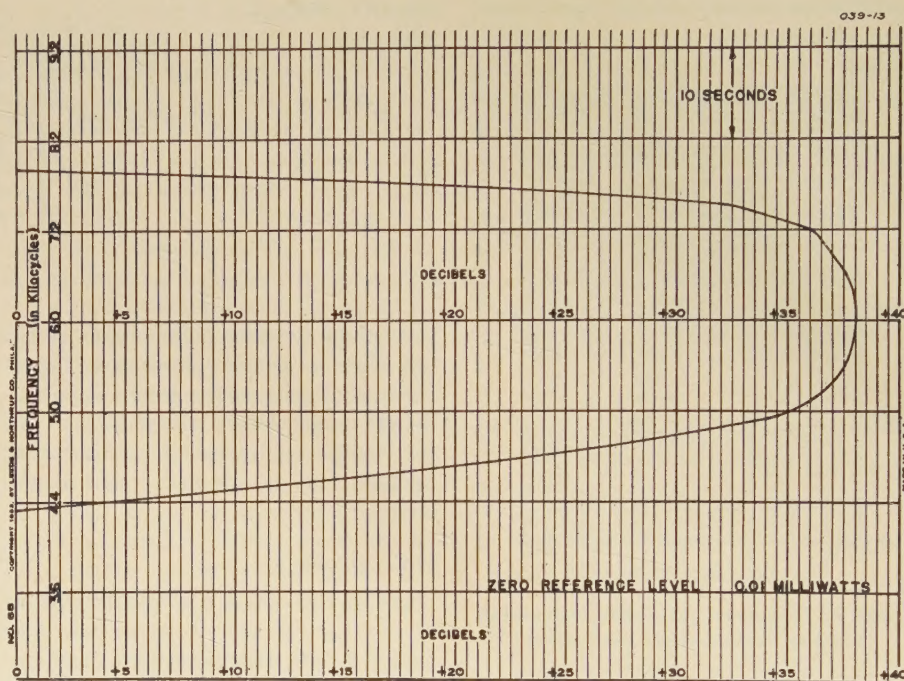


Figure 13. Frequency characteristic of a band-pass filter

Other paper speeds as high as $\frac{3}{4}$ inch per second have been used.

POWER SUPPLY

The power supply for the recorder, exclusive of the plate voltages for the converter and the thyatron assembly, must be very well regulated (figure 6). This is particularly true of the power supply for the detector, the d-c amplifier, and the standard voltage supply, since any variation in their power supply enters directly into the operation of the recorder and will affect its performance. Somewhat greater variations in the power supply of the a-c amplifier would be permissible since these variations do not greatly affect its gain, which is all that need be held constant. Variations in the power supply of the converter will affect only the sensitivity of the recorder.

In order to obtain a power supply which was good enough for satisfactory operation of the recorder, two separate voltage regulators were used. One regulator was of the magnetic type and operated on alternating current. It regulated the 60-cycle power supplied to the power pack-transformer. The other was a gaseous tube regulator which regulated the d-c

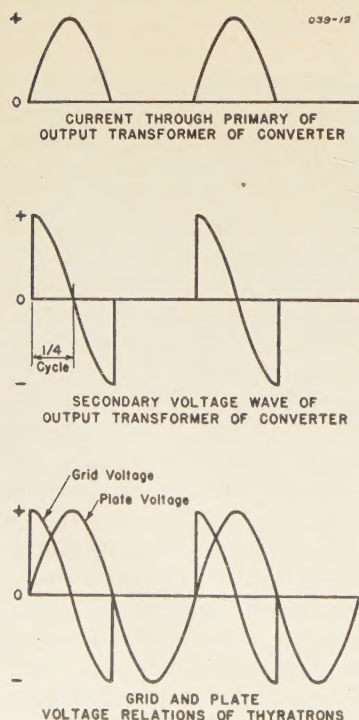


Figure 12. Wave shapes in converter and thyatron assembly circuits

thyratrons (figure 6) which drive the three-lead split-field series reversing motor that is mechanically coupled to the slide-wire attenuator, the tachometer, and the pen.¹ The thyratrons in this recorder perform a control function and as in all forms of control, there is a tendency to hunt. This can be eliminated by temporarily reducing the sensitivity of the control as soon as it starts operating. The sensitivity is temporarily reduced in this instrument by raising the cathode potential of the thyratrons every time they operate, and by allowing them to return to normal only gradually over a period of several cycles. The raising of the cathode potential is equivalent to depressing the grid potential and gives less sensitivity. In the common plate lead is a resistance which develops a drop of several volts upon the operation of either thyatron. This voltage drop charges a capacitor through copper-oxide rectifiers. After the thyratrons are shut off and the plate current goes to zero, the rectifiers discharge the capacitor but only gradually and in

output of the power pack. The d-c output controlled by these two regulators was constant to within 0.1 per cent for a period of 12 hours during which time the 60-cycle supply varied 10 per cent. Calculations and tests indicated that for a recorder accuracy of $\frac{1}{2}$ per cent the d-c power supply should be constant to 0.2

Uses of the Recorder

The uses of this recorder are very varied as can be judged from the following abbreviated list of applications. It may be used for obtaining:

1. Frequency-attenuation characteristics of filters. An example of this is shown in

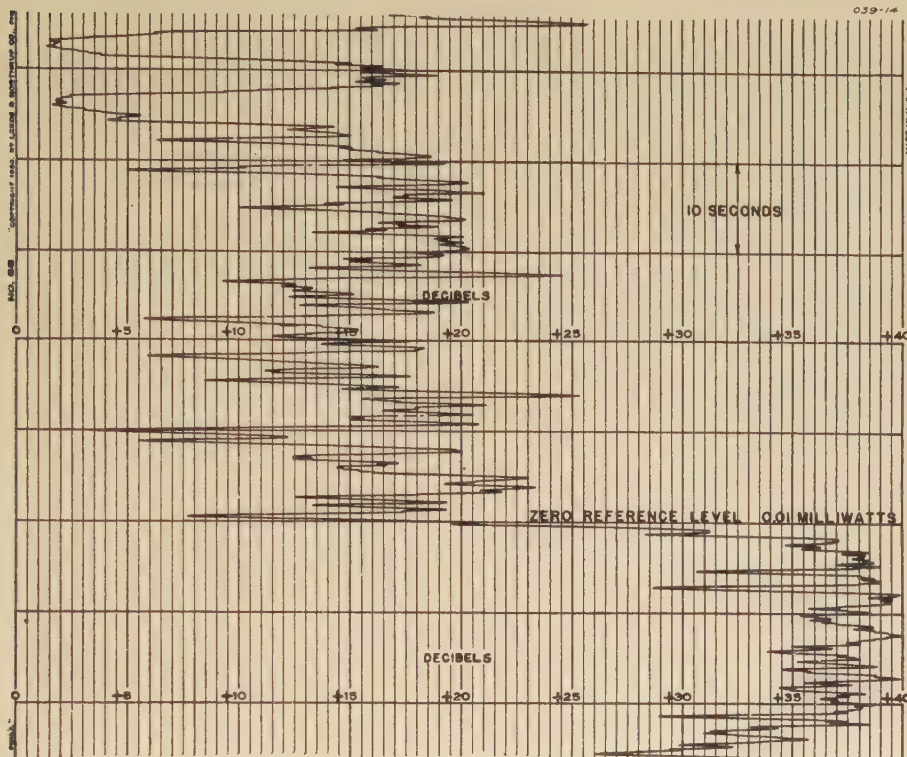


Figure 14. The fluctuating power level of a radio program

per cent, so the regulation obtained by this method is sufficient.

Ease of Operation

While the recorder itself is somewhat complicated, the operation of it is very simple. There are only three adjustments to be made, the "thyatron bias", the "sensitivity", and the "calibration" of the recorder (figure 6). These adjustments do not have to be made every time the recorder is used, but they should be checked occasionally.

In the calibration of the recorder, a high-resistance vacuum thermocouple voltmeter is connected across the input terminals (see figure 6). After the input voltage to the recorder has been adjusted to a definite amount as indicated on the voltmeter, the recorder is adjusted to indicate the corresponding power level by means of the "calibration" dial, which controls the gain of the a-c amplifier.

figure 13 which is the characteristic of a band-pass filter.

2. Frequency-attenuation characteristics of an entire radio set.
3. Frequency-attenuation characteristics of voice channels.
4. Frequency-attenuation characteristics of carrier channels.
5. Frequency-attenuation characteristics of transformers, loud speakers, amplifiers, etc.
6. Measurements of noise levels.
7. Power-level measurements of voice levels occurring in the output of a phonograph record, a radio program, etc. (figure 14).

Since the recorder slide-wire and pen are driven by a fair-size motor, control disks may be mounted on the slide-wire shaft and used for signaling or control purposes (that is, introducing additional attenuation into a filter or a channel to keep the total attenuation or power output within restricted limits).

The slide-wire motor may be used to drive the transmitter of a remote indicator or recorder and thus obtain two records at widely separated points.

Appendix

The Slide-Wire Attenuator

The equations developed here are useful in designing slide-wire attenuators.

If the total attenuation, the characteristic impedance, and the desired accuracy of the attenuator are known, the values of the shunt and series coils can be readily computed. Referring to figures 15A and 15B, let:

- A = total attenuation in decibels
- S = number of sections
- Z_k = characteristic impedance of the attenuator
- Z_b = impedance of each series coil
- Z_a = impedance of first shunt coil
- Z_t = terminating impedance
- E_o = voltage output of one section
- E_o' = voltage output along section
- E_t = voltage input
- n = fraction of a section; ratio of distance between adjacent shunt coil tap and contact to distance between shunt coils. Distances are measured in the output-input direction
- n_{max} = value of n for which the attenuation error is maximum
- D = attenuation error in decibels
- D_{max} = maximum attenuation error in decibels

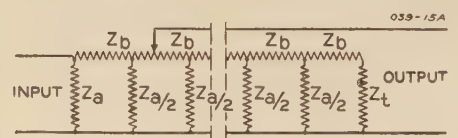


Figure 15A. Schematic diagram of the slide-wire attenuator

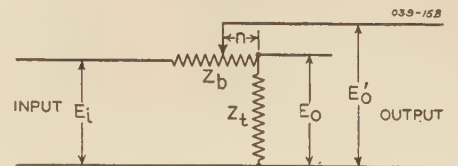


Figure 15B. One section of the slide-wire attenuator

Now the attenuation per section of a slide-wire attenuator is,²

$$-\frac{A}{S} = 20 \log \frac{E_o}{E_i} \quad (1)$$

(all logarithms in this paper are to the base 10).

But

$$\frac{E_o}{E_t} = \frac{Z_t}{Z_t + Z_b}$$

where

$$Z_t = \frac{Z_a Z_k}{Z_a + Z_k} \quad (2)$$

The characteristic impedance of a symmetrical π section attenuator is

$$Z_k = \sqrt{\frac{Z_a^2 Z_b}{2Z_a + Z_b}} \quad (3)$$

Hence

$$Z_b = \frac{2Z_a Z_k^2}{Z_a^2 - Z_k^2} \quad (4)$$

therefore

$$\frac{E_o}{E_i} = \frac{Z_a - Z_k}{Z_a + Z_k} \quad (5)$$

and the attenuation per section can be written,

$$-\frac{A}{S} = 20 \log \frac{Z_a - Z_k}{Z_a + Z_k} \quad (6)$$

the total slide-wire resistance (R_s) is of course

$$R_s = SZ_b \quad (7)$$

If the impedances of the shunt and series coils of the attenuator are assumed to be pure resistances, their values may be obtained from equations 4 and 6 when the total attenuation, the characteristic impedance, and the number of sections are known. The first two are generally known, but the number of sections is not and must be determined from the accuracy requirements in accordance with the following method.

Since the values of the attenuation in decibels at successive shunt coils lie on a curve which is linear, but the variation in attenuation between shunt coils is not linear, the number of sections is determined by the allowable attenuation error of the attenuator. This error can be readily calculated and by using a sufficient number of sections, it can be made negligible.

Referring to figure 15B, the actual attenuation at a fractional position n is:

$$\begin{aligned} -\frac{A}{S}(\text{actual}) &= 20 \log \frac{E_o'}{E_i} = 20 \log \left(\frac{Z_t + nZ_b}{Z_t + Z_b} \right) \\ &= 20 \log \left(\frac{Z_a - Z_k + 2nZ_k}{Z_a + Z_k} \right) \end{aligned}$$

(in terms of Z_a and Z_k)

but from (5)

$$Z_a = Z_k \left[\frac{1 + E_o/E_i}{1 - E_o/E_i} \right]$$

Which gives,

$$-\frac{A}{S}(\text{actual}) = 20 \log \left[n + (1-n) \frac{E_o}{E_i} \right]$$

or

$$-\frac{A}{S}(\text{actual}) = 20 \log \times \left[n + (1-n) \text{antilog} \left(-\frac{A}{20S} \right) \right] \quad (8)$$

The desired attenuation is $(1-n) 20 \log (E_o/E_i)$ or $-(1-n)A/S$.

The attenuation error in one section for a fractional position n is in decibels,

$$D = -(1-n) \frac{A}{S} - 20 \log \times \left[n + (1-n) \text{antilog} \left(-\frac{A}{20S} \right) \right] \quad (9)$$

Therefore, the attenuation error of the slide-wire is independent of the characteristic impedance and dependent only on the decibel range and the number of sections (exclusive of frequency errors).

The point in each section of the slide-wire attenuator at which the error is greatest may be found by differentiating D with respect to n and equating the derivative to zero.

$$\frac{dD}{dn} = +\frac{A}{S} - \frac{20(\log e) \left[1 - \text{antilog} \left(-\frac{A}{20S} \right) \right]}{n + (1-n) \text{antilog} \left(-\frac{A}{20S} \right)}$$

hence

$$n_{\max} = +\frac{20S}{A} \log e - \frac{\text{antilog} \left(-\frac{A}{20S} \right)}{1 - \text{antilog} \left(-\frac{A}{20S} \right)} \quad (10)$$

When this value of $n_{\max}$ is substituted in equation 9, the maximum error ($D_{\max}$) is obtained.

The graphs of figure 9 show the location of the point in each attenuator section at which the attenuation error is greatest ($n_{\max}$) and the value of this error ($D_{\max}$) at

every point in each section. If the value of S/A is greater than $1/4$, the maximum error is small and occurs very close to the midpoint of each section.

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